SPLITTING OF NEUTRON ENERGY LEVELS DUE TO SPIN-ORBIT COUPLING

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ABSTRACT. Due to the spin-orbit interaction the splitting of the energy levels of neutrons moving in the potential $V = V(r) + \alpha \frac{\hbar^2}{M^2C^2} \frac{1}{r} \frac{dV}{dr} \vec{\sigma} \vec{l}$ is calculated for p-states for atomic mass 200. The effect of the spin-orbit coupling is calculated by the perturbation method with the unperturbed wave functions as obtained by Ghosh and Sil (in course of publication) by the technique of Lanczos.

For the calculation of energy levels of neutrons moving in the nucleus we choose the nuclear potential as of the form

$$V = V(r) + \alpha \frac{\hbar^2}{M^2C^2} \frac{1}{r} \frac{dV}{dr} \vec{\sigma} . \vec{l}$$

where $V(r) = -V_0[1 + e^{(r-R)/a}]^{-1}$ and $\alpha = -\frac{15}{2}$

The constant α is the same as taken by Fermi (1954) for polarization of high energy protons scattered by nuclei. The operator $\vec{\sigma} . \vec{l}$ in the above expression can be replaced by its eigen values (cf Mayer and Jensen, 1955). Thus

$$V = V(r) + \alpha \frac{\hbar^2}{M^2C^2} \frac{1}{r} \frac{dV}{dr} l \qquad \text{for } j = l + \tfrac{1}{2}$$

$$= V(r) - \alpha \frac{\hbar^2}{M^2C^2} \frac{1}{r} \frac{dV}{dr} (l+1) \quad j = l - \tfrac{1}{2}$$

We shall consider the splitting of the energy levels due to the spin-orbit coupling term by the usual perturbation method; to obtain the unperturbed solutions, we neglect this term in the Schrodinger equation and solve it by a method due to Lanczos (1934). This procedure is previously (Chosh and Sil, 1960) adopted in obtaining the single particle bound states of neutrons moving in the

potential $V(r) = -V_0[1+e^{(r-R)/a}]^{-1}$. The separation of the perturbed state from the unperturbed level is given by

$$\Delta E = \int \psi^* H \psi d\tau$$

when H represents the perturbation and ψ is the unperturbed wave function (normalized)

Since the operator $\vec{\sigma} \cdot \vec{l}$ in the perturbation has been replaced by its eigen values we need consider only the radial part of the integral The range of the above integration is split up into four regions which are as follows . $I. 0 \leqslant r \leqslant r_1 = R - a \log_e 9$, II $r_1 \leqslant r \leqslant R$, III. $R \leqslant r \leqslant r_2 = R + a \log_e 9$ and IV. $r_2 \leqslant r \leqslant \infty$. The limits of r_1 and r_2 are so chosen that the nuclear potential drops respectively to 9/10 and 1/10 of its value at $r = 0$

The unperturbed wave functions of the neutron in the four regions are :

$$\psi_I = j_l(\gamma x)$$

$$\psi_{II} = \frac{1}{x}[Ae^{x\gamma'} F^+(q) + A^* e^{-x\gamma'} F^-(q)]$$

$$\psi_{III} = \frac{1}{x}[A^+ e^{+k'x} F^+(p) + A^- e^{-k'x} F^-(p)]$$

and $$\psi_{IV} = C h_l^{(1)}(ikx)$$

where j_l and h_l are respectively the spherical Bessel and Hankel function of order l, F's are polynomials in p or q, further

$$p = \frac{1}{8}(9e^{(R-r)/a} - 1)$$

$$q = \frac{1}{8}(9e^{(r-R)/a} - 1)$$

$$\gamma = \sqrt{\lambda^2 - k^2}, \qquad \gamma' = \sqrt{\lambda^2 - k''^2}$$

where $$\lambda^2 = \frac{2M}{\hbar^2} V_0 a^2 ; \quad k^2 = \frac{2M}{\hbar^2} |E| a^2$$

$$k'^2 = k^2 + \frac{l(l+1)}{2} a^2 \left[\left(\frac{1}{R}\right)^2 + \left(\frac{1}{R + a \log_e 9}\right)^2\right]$$

$$k''^2 = k^2 + \frac{l(l+1)}{2} a^2 \left[\left(\frac{1}{R}\right)^2 + \left(\frac{1}{R - a \log_e 9}\right)^2\right]$$

The constants A, A^*, A^+, A^- and C of the unperturbed wave functions are determined by the usual matching of ψ_I, ψ_{II} and their derivatives at $r = r_1$, ψ_{II}, ψ_{IV} and their derivatives at $r = r_2$ and finally ψ_{II}, ψ_{III} and their derivatives at $r = R$. Since the spin-orbit interaction is only a surface effect, in this paper we have evaluated the perturbation integral only in the two short ranges r_1 to R and R to r_2 near the boundary. In these two short ranges we have evaluated the integral numerically applying Simpson's rule

The parameters in the central part of the potential are $V_0 = 52$ Mev, $R = (1.15A^{1/3}+0.4)10^{-13}$cm, $a = 0.57 \times 10^{-13}$ cm. The calculated values of the separation of the levels for $j = 1/2$ and $j = 3/2$ for atomic mass 200 are shown in Table I.

TABLE I

State	Separation between the states $j = 1/2$ and $j = 3/2$
$1p$	0.69 MeV
$2p$	0.70 MeV
$3p$	0.83 MeV

In conclusion we may mention that for atomic mass 208 Ross *et al.* (1956) have obtained a separation of nearly 0.9 Mev. between $3p_{\frac{1}{2}}$ and $3p_{3/2}$ states. In our case for $A = 200$ we have obtained a separation of 0.8 MeV.

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